HYPERBOLIC DYNAMICS AND THE GEOMETRY OF INVARIANT SETS
MWF 3:00PM – 3:50PM

INSTRUCTOR: WILLIAM YESSEN

1. Introduction

1.1. **Description.** In this class we will study dynamical systems given by iteration of smooth (and typically invertible) maps on smooth manifolds which are characterized by the presense of contracting and expanding directions for the derivative. Such systems are called *hyperbolic*.

Hyperbolic dynamical systems constitute a vast and very well developed branch of the modern dynamical systems theory. While most of the theory of hyperbolic dynamics had been developed in the 20th century, many interesting problems are still unsolved, and many nontrivial applications emerge regularly (e.g. in mathematical physics and statistical mechanics, celestial mechanics, biology, electrical engineering, quantum mechanics, just to name a few).

Generally when studying a dynamical system, one is interested in the structure of the orbits obtained by iterating a map with given initial conditions. In the case of smooth systems (i.e. iterating smooth maps on smooth manifolds), one may seek to first understand the local behavior by locally linearizing the system, and then extend the picture semi-locally or even globally. Given the expanding and the contracting properties of the differential in a hyperbolic system, one has an essentially complete local picture, dominated by exponential convergence/divergence, and in many cases it is possible to obtain the global picture from this. This exponential behavior, often coupled with "folding" and recurrence, leads to what appears to be an effectively random system emerging from a completely deterministic one. This complex phenomenon of *deterministic chaos* appears both, in the context of pure mathematics and in many physical systems (some mentioned above).

Typically a system given by a smooth map T acting on a manifold M is not hyperbolic on the entire phase space M, but on a subset Λ. This set is precisely where most interesting dynamics occurs, and the complexity of the entire system stems from the dynamics of $T|_\Lambda$. Thus in order to quantify the effects that $T|_\Lambda$ has on the entire system, one studies geometric, topological, measure-theoretic and fractal-dimensional properties of Λ. Typically Λ will have rich geometric properties, so that the study of hyperbolic dynamics involves an elaborate interplay of ideas from geometry, topology, analysis, and geometric measure theory.

Thus the aim of this class is to give a comprehensive survey (complete with proofs of the main results) of hyperbolic dynamics, together with a detailed account of the techniques used in the study of such systems. We will also try to give examples of some nontrivial applications (level of detail will depend on how much time we have).

1.2. **Prerequisites.** Rigorous understanding of general topology and analysis (measure theory and integration) would be very helpful. Some might find helpful to have a general understanding of smooth manifolds, but this isn't required (we'll cover what we need as we go along). While we do not assume prior acquaintance with dynamical systems, this class is not an intro to dynamics; instead all necessary concepts will be introduced as needed.

2. Organization

The class will meet on MWF 3:00pm – 3:50pm. There won't be any exams but will be occasional homework. The grade will be based on the homework and participation in class. As far as office hours: I'll be in my office most of the time, so just pop in (or arrange a meeting in advance).

3. Study Material

There won't be any required texts, but for reference and further reading I'd recommend the following for some parts of the course:

- *Introduction to the modern theory of dynamical systems* by Katok and Hasselblatt,
- *Introduction to dynamical systems* by Brin and Stuck,
- *Ergodic theory, hyperbolic dynamics and dimension theory* by Barreira
- *Hyperbolicity and sensetive chaotic dynamics at homoclinic bifurcations* by Palis and Takens.

I will also occasionally provide extended lecture notes.

Date: July 29, 2015.